Wave Propagation in Turbulent Sea Water†

Y. Lacroix[1], D. Leandri[1], and V. Nikishov[2]

[1]Laboratoire des Systémes Navals Complexes,
Toulon, France

[2]Institute of Hydromechanics, National Academy of Sciences
Zhelyabov St., 8/4, 03680 MSP, Kyiv, Ukraine
E-mail: vinihm@gmail.com

Characteristics of light wave propagating in a turbulent sea water are determined by fluctuation spectrum of optical refractive index n. The behavior of the spectrum is controlled by fluctuations of temperature and salinity. Acoustic scattering from oceanic microstructure is due to sound speed and density fluctuations that in turn also depend on the temperature and salinity fluctuations. The work gives a short description of the spectrum model of fluctuations of optical refractive index depending on dissipation rates of component fluctuations, as well as the dissipation rates of turbulent energy, which describes both inertial-convective and viscous-convective subranges. Others values, like n (density, conductivity, sound speed), can be also considered. Some statistical characteristics of the light wave for real distributions of T and S in depth were calculated by the methods of geometric optics and smooth perturbations using the developed spectrum model. It was shown that the above spectrum peculiarities lead to considerable changes of statistical characteristics of propagating waves, in particular, level fluctuations, correlation function of the level fluctuation, structural function of the phase.

* * *

Introduction

When acoustic and electromagnetic waves propagate in the medium with random inhomogeneities one can observe changes in wave amplitudes, their phases, arrival angles, etc. These changes are of random character. Turbulence is one of the main causes of the above inhomogeneities [1, 2]. It determines, for example, the existence of random temperature and humidity fluctuations in the stratified atmosphere, as well as those of the refractive index.

Theoretical principles of studying characteristics of the waves which propagate in the turbulent atmosphere have been developed in [1–4]. As a rule Corrsin – Obukhov's model of the spectrum

†Received 27.12.2010

of turbulent fluctuations [5] is used in these works. In the above model spectrum of temperature fluctuations is described by the expression $E_T \propto \kappa^{-5/3}$ in the inertial-convective subrange, and for large wave numbers this spectrum attenuates following the exponential law. Here, κ is the wave number. The results of computation of statistical wave characteristics performed using the above expression for the spectrum of temperature fluctuations were compared with experimental data of measurements, and this comparison has shown a rather good agreement.

In contrast to the atmosphere, fluctuations of the light wave characteristics in the sea medium are determined both by the temperature and salinity fluctuations. The temperature and salinity fluctuation spectra in the inertial-convective subrange, as well as in the atmosphere are subjected to the Obukhov–Corrsin's law $E_T \propto \kappa^{-5/3}$ and $E_S \propto \kappa^{-5/3}$. The interval of wave numbers reached by these spectra is bounded from above by the effects of temperature conductivity and salt diffusivity, respectively. Velocity fluctuation energy spectrum in water medium sharply diminishes at $\kappa > \kappa_K = 1/\eta$ where $\eta = (\nu^3/\varepsilon)^{1/4}$ is Kolmogorov's scale which characterizes the effect of viscosity; ε is the rate of dissipation of turbulent energy; ν is the molecular viscosity. The water medium distinction is that the coefficient ν exceeds considerably molecular coefficients of temperature conductivity χ_T and salt diffusion χ_S: $\nu \gg \chi_T \gg \chi_S$. The result is that, in contrast to the velocity fluctuation spectrum, the fluctuation spectra of temperature and salinity at $\kappa > \kappa_K$ decrease smoothly down to very small scales, where the processes of temperature conductivity and salt diffusion play an important role. In these viscous-convective subranges the spectra E_T and E_S are in inverse proportion to the wave number (Batchelor's law): $E_T \propto \kappa^{-1}$, $E_S \propto \kappa^{-1}$ [5, 6]. Extents of the subranges are determined by the ratios of molecular coefficients of transfer, that is Prandtl number $\mathrm{Pr} = \nu/\chi_T$ and Schmidt number $\mathrm{Sc} = \nu/\chi_S$. Thus, the spectral subranges arise at small scale in water. Their appearance is caused by difference between molecular transfer coefficients on the one hand, ν and on the other hand, χ_T, χ_S. Hill [6] was the first who concerned the above peculiarities of temperature and salinity fluctuation spectra in respect of the problems of light propagation in the turbulent water medium.

The effect of temperature fluctuations on the efficient scattering cross-section [7, 8] is usually considered in the models describing scattering of the acoustic waves by inhomogeneities of sea medium. The density effect on the efficient scattering cross-section is allowed for in more exact models. There arises a necessity of considering not only temperature but also salinity fluctuations. The authors of [9] present the data of measuring the microstructure parameters in the tidal channel, and they were assumed as a basis of verification of measurement results of backscattering of the acoustic waves by turbulent microstructure within the billows. Note, the one-dimensional spectrum without term describing co-spectrum fluctuations of temperature and salinity was used to analyze the results. It was concluded that the existing models underestimated the intensity of acoustic scattering from turbulent microstructure, especially when scattering is determined by viscous-convective subrange of the turbulent spectrum. Theoretical analysis of sound backscattering by microstructure peculiarities of sea medium is performed in [10]. The work indicates to the important part of co-spectrum of temperature and salinity fluctuations when calculating the efficient scattering cross-section. Following the work [11] the author has estimated the maximum effect of the above spectrum using the known spectral inequality [12] $\phi_{TS} \leq (\phi_T \phi_S)$, where ϕ_T, ϕ_S are one-dimensional spectra of temperature and salinity fluctuations, respectively, ϕ_{TS} is a one-dimensional co-spectrum of temperature and salinity fluctuations. It was shown that the contribution of salinity fluctuations to the value of the efficient scattering cross-section may be equal (and even more) to contribution of temperature fluctuations, especially for frequencies above 100 kHz, in case when salinity determines the fields of density. An expression was obtained in [13] for the efficient scattering cross-section by the ocean turbulent microstructure, the effect of changes both in temperature and salinity on sound speed and density being allowed for in this expression. The authors used the above model of the co-spectrum of temperature and salinity fluctuations. They also considered another (linear) model of the co-spectrum based on the analysis of TS-gradients for small spatial scales

$$\phi_{TS} = \frac{\phi_T}{\delta} + \frac{\delta \phi_S}{r_\varepsilon},$$

where

$$r_\varepsilon = \frac{\chi_T}{\chi_S}, \qquad \delta = \frac{\partial T/\partial z}{\partial S/\partial z}.$$

In this case the co-spectrum is determined by contributions of the temperature and salinity fluctuations, and the value of the efficient scattering cross-section depends on the sign δ. It is evident that the spectrum is not equal to zero in the interval $\kappa_B^S > \kappa > \kappa_B^T$ (κ_B^T, κ_B^S are the wave numbers corresponding to the end of the viscous-convective subrange of spectra of the temperature and salinity fluctuations, respectively), though at $\kappa > \kappa_B^T$ the co-spectrum value should quickly attenuate. This leads to overestimation of the value of the efficient scattering cross-section.

Basing on the results of the work [5], the authors of [14] have obtained an expression describing the spectrum of sea water density fluctuations, including the co-spectrum in the case when the above fluctuations are determined by those of temperature and salinity. It was supposed that peculiarities of the spectrum of density fluctuations in dissipative interval are mainly connected with different decay of the temperature and salinity fluctuations in cascade process of energy transfer to the large wave numbers. Such an approach is based on the fact that the transfer processes in the dissipative region are not determined by velocity fluctuations, as it takes place in the inertial subrange, but by the deformation rate $\gamma = (\varepsilon/\nu)^{1/2}$, which remains unchanged for the both scalar values. The same approach was used in [15] to obtain the expression for the co-spectrum of temperature and salinity fluctuations, and a comparison was made for the results of calculation of the scattering cross-section of the acoustic wave obtained when using the proposed model and using the already described models of the spectrum. The authors emphasize the importance of allowance for the co-spectrum in the problems of scattering of the high-frequency acoustic waves by microstructure peculiarities of the sea medium. The effect of the co-spectrum of temperature and salinity fluctuations on fluctuation spectrum of electrical conductivity was analyzed in the works [11, 16]. The model computations have shown that this effect is rather essential, and at compensation of contributions of temperature and salinity fluctuations it can result in the sharp decrease of the spectral level.

Basing on the Corrsin – Pao's model [17, 18], the model of fluctuation spectrum of the scalar value (the optical refractive index here) which is determined by temperature and salinity fluctuations, is generalized in [19], where the expression was obtained which describes the inertial-convective and viscous-convective subranges. It was shown by parametrization of the averaged fields that the peculiarities of the spectrum behaviour in the region of large wave numbers depends on contribution of the temperature and salinity fluctuations to fluctuations of the refractive index.

Peculiarities of statistical characteristics of a plane light wave propagating in the turbulent sea medium are analyzed in the given work basing on the expression obtained in [19] which describes the spectrum of fluctuations of the optical refractive index. The work structure is as follows. Section 1 describes in brief the procedure of obtaining the fluctuation spectrum of the optical refractive index of sea water. Section 2, basing on the method of geometrical optics, presents results of calculations of some statistical characteristics of a plane light wave propagating in the turbulent sea medium. Section 3 is dedicated to analysis of peculiarities of the propagating light wave on the basis of the method of smooth perturbations.

1. Spectrum of Turbulent Fluctuations of the Optical Refractive Index

Let us suppose that turbulence is a local-homogeneous and isotropic one. Note, that statistical behaviour of multicomponent mixture in the local-homogeneous and isotropic turbulence was studied in a number of works as applied to the problems of burning, operation of chemical reactors, etc. In work [20] the expressions were obtained which describe the spectrum of the mixture concentration fluctuations with the availability of the first-order chemical reaction. The concentration spectrum behaviour was studied in the range of large wave numbers for the case of equality of molecular diffusion coefficients of the mixture components. The same case was considered in the work [21], where they had studied the fluctuation spectrum of the mixture which consisted of two components with identical diffusion coefficients. The Lagrangian stochastic model was used in the work [22] to calculate the correlation between two primarily segregated components with the same molecular diffusion coefficients. The direct numerical modeling of the diffusion of passive scalar values with different molecular diffusion coefficients in the steady isotropic turbulence was performed in the works [23, 24]. Main attention is given in these works to studying the role of different classes of triad interactions in the cascade process of energy transfer. As a rule, the Schmidt numbers $\mathrm{Sc} = \nu/\chi_S$, belonging to separate mixture components did not exceed a unit in these works and differed from one another no more than 4 times. It has been demonstrated that the scalar fields, being identical in the initial moment, lose gradually their correlation as affected by diffusion, and the effect of different diffusion coefficients is first manifested in the range of small scales, and then extends to the range of large scales.

Consider first the temperature fluctuation spectrum. Let us start from the following spectral equation [5, 19, 25] which describes changes of the spectral distribution of intensity of temperature fluctuations in time.

$$\frac{\partial}{\partial t}E_T(t,\kappa) - W_T(t,\kappa) = -2\chi_T\kappa^2 E_T(t,\kappa), \tag{1}$$

where $E_T(t,\kappa)$ is a three-dimensional spectral density, function $W_T(t,\kappa)$ allows for the effect of turbulent mixing and satisfies the equality

$$\int_0^\infty W_T(t,\kappa)d\kappa = 0.$$

One of the ways of solution of the Eq. (1) is the introduction of scalar spectral flux function $F_T(t,\kappa)$, determined by expression $W_T = -\partial F_T/\partial\kappa$, and establishing of the relation between $F_T(t,\kappa)$ and $E_T(t,\kappa)$. One of such models, offered for a steady case, is the Corrsin – Pao model [17, 18], in which it is supposed that the spectral element characterized by the wave number κ, is transferred to a larger wave number $\kappa + d\kappa$ during the time $d\tau$. Here $\tau(\kappa)$ is a time scale characteristic of the given spectral interval. When introducing the conventional velocity $B(\kappa)$, determining the transfer of the spectral element through the wave number κ, $B = d\kappa/d\tau \cong \kappa/\tau(\kappa)$, the scalar spectral flux function $F_T(t,\kappa)$ may be represented as $F_T(t,\kappa) = B(\kappa)E_T(\kappa)$.

The considered model is based on a supposition of locality of energy transfer from smaller to larger wave numbers. The model validity is confirmed by results of modeling of the process of energy transfer in homogeneous turbulence by numerical solution of Navier – Stocks equations, that is considered in the space of wave numbers, on the basis of triad interaction of spectral components. These interactions, with a condition $\kappa_1 = \kappa_2 + \kappa_3$ fulfilled for them, where κ_i $(i = 1, 2, 3)$ are wave numbers of spectral components, are the main process of energy transfer from smaller to larger wave numbers [26]. It was shown [23, 27] , that in inertial subrange the main contribution to